GRAVITATIONAL WAVES FROM ACCRETING NEUTRON STARS [a]

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We show that accreting neutron stars in binary systems or in Landau-Thorne-Zytkow objects are good candidates for continous gravitational wave emission. Their gravitational radiation is strong enough to be detected by the next generation of detectors having a typical noise of 10^{-23} Hz$^{-1/2}$.

1 Introduction

A crude estimate of the gravitational luminosity of an object of mass M, mean radius R and internal velocities of order V can be derived from the quadrupole formula:

$$L \sim \frac{c^5}{G}\, s^2 \left(\frac{R_{\rm s}}{R}\right)^2 \left(\frac{V}{c}\right)^6 \;, \tag{1}$$

where $R_{\rm s} := 2GM/c^2$ is the Schwarzschild radius associated with the mass M and s is some asymmetry factor: $s = 0$ for a spherically symmetric object and $s \sim 1$ for an object whose shape is far from that of a sphere. According to formula (1), the astrophysical objects for which $s \sim 1$, $R \sim R_{\rm s}$ and $V \sim c$ may radiate a fantastic power in the form of gravitational waves: $L \sim c^5/G = 3.6 \times 10^{52}$ W, which amounts to 10^{26} times the luminosity of the Sun in the electromagnetic domain!

A neutron star (hereafter NS) has a radius quite close to its Schwarzschild radius: $R \sim 1.5 - 3\,R_{\rm s}$ and its rotation velocity may reach $V \sim c/2$ at the equator, so that they are a priori valuable candidates for strong gravitational emission. The crucial parameter to be investigated is the asymmetry factor s. It is well known that a uniformly rotating body, perfectly symmetric with respect to its rotation axis does not emit any gravitational wave ($s = 0$). Thus in order to radiate gravitationally a NS must deviate from axisymmetry. Moreover, CW emission is possible only if the NS accreets angular momentum from an angular momentum reservoir.

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Low Mass X-ray Binary systems (LMXB) and High Mass X-rays Binary systems (HMXB) are a good examples of a NS coupled with an angular momentum reservoir. These systems are formed by a NS and an ordinary companion. If the two stars are close enough, the NS accretes matter (and angular momentum) from the companion and consequently can radiate CW if its axisymmetry is broken.

The fate of such a system depends on the mass of the companion. For the HMXB for which the companion is a massive star ($\geq 8M_\odot$) and consequentely the life time is quite short ($\approx 10^6$ yr) the companion evolves until when the nuclear fuel is exhausted and becomes a supernova. If the binary system is not disrupted by the explosion, the outcome is a binary pulsar. PSR B1913+16 is a good illustration of this scenario. If the companion is kicked away by the explosion, then the outcome is an isolated pulsar.

If the mass of the companion is lower than $1\,M_\odot$ (LMXB), the NS is spun up by the accretion of matter and angular momentum and, provided the axisymmetry is broken, the NS radiates gravitational waves steadily if the accreted angular momentum is evacuated via gravitational radiation. The light companion is evaporated by the electromagnetic emission of the NS and the final outcome is an isolated millisecond pulsar.

In the intermediate case (mass of the companion between $1M_\odot$ and $8M_\odot$) the final state is a binary system formed by a white dwarf and a millisecond pulsar. The important point is that by measuring the period modulation of the pulsar in a binary system it turns out to be possible to measure the mass of the NS. The mass of the NS is a fundamental parameter as will be explained later.

Landau-Thorne-Zytkow objects (LTZO) constitute another example of a NS coupled with an angular momentum reservoir. These objects, introduced by Landau [1] to explain the stellar source of energy, have been discussed in details by Thorne and Zytkow [2]. They look as ordinary red supergiant stars, the main difference being the core which is a NS instead of being white-dwarf like. The origin of these objects (if they exist) is beleived to be a HMXB during the phase in which the NS is orbiting into the envelop of the companion. Another possible origine are *aborted supernovae*, i.e. supernovae for which the explosion is not strong enough to eject the envelop. The NS forming the core of these objects accrets matter from the envelop at the maximum rate, i.e. at the Eddington limit : $10^{-8}\,M_\odot\,\mathrm{yr}^{-1}$. If some amount of angular momentum is stored in the envelop, the NS is spun up by this accretion. The life of a LTZO is about 10^8 yr, until the mass of the NS reaches the maximum value M_{max} and the NS collpases into a black hole. It must be noticed that the mass of NS, during its life, varies between the values of the mass at which the NS is

born to the value of $M_{\max}$ that depends on the equation of state (EOS).

2 Symmetry Breaking Mechanisms

As already said, gravitational waves are radiated by a rotating NS only if its axisymmetry is broken. Two distinct classes of symmetry breaking mechanisms exist: The axisymmetry can be broken spontaneously (via some kind of instability of the NS) or the axisymmetry can be broken via some external mechanism. Both cases are pertinent to what follows and therefore will be discussed in some detail.

A rotating NS can break spontaneously its axial symmetry if the ratio of the rotational kinetic energy T to the absolute value of the gravitational potential energy, $|W|$, exceeds some critical value. When the critical threshold $T/|W|$ is reached, two kinds of instabilities may drive the star into the non-axisymmetric state:

1. the *Chandrasekhar-Friedman-Schutz instability* (hereafter *CFS instability*) [3],[4],[5] driven by the gravitational radiation reaction.

2. the viscosity driven instability [6].

Let us recall some classical results from the theory of rotating Newtonian homogeneous bodies. It is well known that a self-gravitating incompressible fluid rotating rigidly at some moderate velocity takes the shape of an axisymmetric ellipsoid: the so-called *Maclaurin spheroid*. At the critical point $T/|W| = 0.1375$ in the Maclaurin sequence, two families of triaxial ellipsoids branch off: the *Jacobi ellipsoids* and the *Dedekind ellipsoids*. The former are triaxial ellipsoids rotating rigidly about their smallest axis with respect to an inertial frame, whereas the latter have a fixed triaxial figure in an inertial frame, with some internal fluid circulation at constant vorticity (see ref. [7] or [8] for a review of these classical results). The Maclaurin spheroids are dynamically unstable for $T/|W| \geq 0.2738$. Thus the Jacobi/Dedekind bifurcation point $T/|W| = 0.1375$ is dynamically stable. However, in presence of some dissipative mechanism such as viscosity or gravitational radiation (CFS instability) that breaks the circulation or angular momentum conservation, the bifurcation point becomes secularly unstable against the $l = 2, m = 2$ "bar" mode. Note also that a non-dissipative mechanism such as a magnetic field with a component parallel to the rotation axis breaks the circulation conservation [9] and may generate a spontaneous symmetry breaking. If one takes into account only the viscosity, the growth of the bar mode leads to the deformation of the Maclaurin spheroid along a sequence of figures close to some Riemann

S ellipsoids[b] and whose final state is a Jacobi ellipsoid [11]. On the opposite, if the gravitational radiation reaction is taken into account but not the viscosity, the Maclaurin spheroid evolves close to another Riemann S sequence towards a Dedekind ellipsoid [12].

The CFS instability is due to the coupling between the degrees of freedom of the star and gravitational waves: the star can loose angular momentum (and kinetical energy) via gravitational radiation. The formation of waves on the sea when the wind blows is due to an analougous mechanism: in the frame of reference of the wind, the water looses momentum because of its coupling with the atmosphere. Two conditions must be fulfilled to allow for the growth of the CFS instability: (i) the phase velocity of the pertubation must be less then the rotation velocity at the equator of the star, (ii) the viscosity must be less than a threshold value μ_{crit}.

The first condition is always met: it turns out that the phase velocity of the gravity waves (the so-called f modes) is $\propto l^{-1/2}$, where l is the "quantum" number of the wave in the harmonic functions expansion (all that in a complete analogy with the sea waves). On the contrary, the second condition is hardly fulfilled: the dumping effect due to the viscosity grows as l^2. Therefore, taking into the account the viscosity of nuclear matter, only the mode $l = 2$ can survive. Recent computations [13] show that this kind of instability can exist only during a short period in the life of the star: in fact if the NS is too hot (resp. too cool), the bulk viscosity (resp. the shear viscosity) inhibits the instability. Actually the interior of a NS is more complicated: it is superfluid and type 2 superconductor. Superfluid vortices are coupled with magnetic fluxoides via their own magnetic field. Vortices and fluxoides are strongly pinned in the solid crust of the star. All that results in an effective viscosity higher than the one computed in the absence of magnetic field. Moreover, any mechanism that tends to rigidify the rotation of star (for example the magnetic field) acts against the CFS instability. From the above it turns out that the CFS instability seems to be very unlikely.

The viscosity driven instability seems to be more promising: in fact, its rising time decreases when the viscosity increases. The physical mechanism of this instability is very simple: consider the rotational kinetical energy of the NS at fixed angular momentum L: $T = L^2/I$ where I is the moment of inertia with respect to the rotation axis. The kinetical energy T decreases if I increases.

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[b]The *Riemann S* family is formed by homogeneous bodies whose fluid motion can be decomposed into a rigid rotation about a principal axis and a uniform circulation whose vorticity is parallel to the rotation vector. Maclaurin, Jacobi and Dedekind ellipsoids are all special cases of Riemann S ellipsoids (for more details, cf. Chap. 7 of ref. [7] or Sect. 5 of ref. [10]).

It turns out that for a large enough L, the total energy of the star (sum of the kinetical and gravitational energy) decreases when I increases. The natural way to increase I is to let the configuration to be tri-axial. It is worth to note that the final stellar configuration is again an equilibrium configuration (in a rotating frame). The transition between Maclaurin and Jacobi configurations is a real Landau phase transition of the second order as was showed by Bertin and Radicati[14]. The reader can find more details in our lecture on the subject at Les Houches School[15].

The main problem is that this instability can work, as already said, only if the NS rotates fast enough. The maximun angular velocity of a rigidly rotating star is achieved when the velocity at the equator is equal to the Keplerian velocity. It turns out that if the EOS of the fluid forming the star is too soft, the Keplerian velocity is less than the critical velocity for which the axisymmetry breaks. For a polytropic EOS ($P \propto \rho^\gamma$) and in the Newtonian theory, γ must be greater than $\gamma_{\mathrm{crit}} = 2.238$ (ref. [16], [17], [18]).

3 Results for realistic equations of state

Recently, we have generalized the above results to the existing "realistic" EOS in a General Relativistic frame[17]. Table 1 shows the results: among the 12 EOS taken under consideration, five are stiff enough to allow for the transition toward a 3-D configuration. In table 1, the EOS are labeled by the following abbreviations: PandN refers to the pure neutron EOS of Pandharipande[19], BJI to model IH of Bethe & Johnson[20], FP to the EOS of Friedman & Pandharipande[21], HKP to the $n_0 = 0.17 \, \mathrm{fm}^{-3}$ model of Haensel et al. [22], DiazII to model II of Diaz Alonso[23], Glend1, Glend2 and Glend3 to respectively the case 1, 2, and 3 of Glendenning EOS[24], WFF1, WFF2 and WFF3 to respectively the $\mathrm{AV}_{14} + \mathrm{UVII}$, $\mathrm{UV}_{14} + \mathrm{UVII}$ and $\mathrm{UV}_{14} + \mathrm{TNI}$ models of Wiringa et al. [25], and WGW to the $\Lambda_{\mathrm{Bonn}}^{00} + \mathrm{HV}$ model of Weber et al. [26].

From the above results it appears that only NSs whose mass is larger than $1.64 \, M_\odot$ meet the conditions of spontaneous symmetry breaking via the viscosity-driven instability. The above minimum mass is quite below the maximum mass of a fast rotating NS for a stiff EOS ($3.2 \, M_\odot$ [27]). Note that the critical period at which the instability happens ($P = 1.04$ ms) is not far from the lowest observed one (1.56 ms). The question that naturally arises is: do these heavy NSs exist in nature ? Only observations can give the answer; in fact, the numerical modelling of a supernova core and its collapse [28] cannot yet provide us with a reliable answer. The maximum critical rotation period (1.2 ms) at which the the instability appears is compatible with the rotation period of the fastest known pulsar (1.56 ms); moreover the age of these pulsar

EOS	$M_{\mathrm{max}}^{\mathrm{stat}}$ $[M_\odot]$	$M_{\mathrm{max}}^{\mathrm{rot}}$ $[M_\odot]$	P_{K} [ms]	P_{break} [ms]	$H_{\mathrm{c,break}}$	M_{break} $[M_\odot]$
HKP	2.827	3.432	0.737	1.193	0.168	1.886
WFF2	2.187	2.586	0.505	0.764	0.292	1.925
WFF1	2.123	2.528	0.476	0.728	0.270	1.742
WGW	1.967	2.358	0.676	1.042	0.170	1.645
Glend3	1.964	2.308	0.710		stable	
FP	1.960	2.314	0.508	0.630	0.412	2.028
DiazII	1.928	2.256	0.673		stable	
BJI	1.850	2.146	0.589		stable	
WFF3	1.836	2.172	0.550	0.712	0.327	1.919
Glend1	1.803	2.125	0.726		stable	
Glend2	1.777	2.087	0.758		stable	
PandN	1.657	1.928	0.489		stable	

Table 1: Neutron star properties according to various EOS: $M_{\mathrm{max}}^{\mathrm{stat}}$ is the maximum mass for static configurations, $M_{\mathrm{max}}^{\mathrm{rot}}$ is the maximum mass for rotating stationary configurations, P_{K} is the corresponding Keplerian period, P_{break} is the rotation period below which the symmetry breaking occurs, $H_{\mathrm{c,break}}$ is the central log-enthalpy at the bifurcation point and M_{break} is the corresponding gravitational mass. The EOS are ordered by decreasing values of $M_{\mathrm{max}}^{\mathrm{stat}}$.

spans between 10^7 and 10^9 yr.

The real problem is the minimum mass, $1.64 M_\odot$, for the triaxial instability to develop. This is not in very good agreement with the measured masses (all in binary systems)[29], except for PSR J1012+5307 which appears to be a heavy NS: $1.5\,M_\odot < M < 3.2\,M_\odot$ [30]. Four NS masses (all in binary radio pulsars) are known with a precision better than 10% and they turn out to be around $1.4\,M_\odot$ [29]. Among the X-ray binary NSs, two of them seem to have a higher mass: 4U 1700-37 and Vela X-1 ($1.8 \pm 0.5\,M_\odot$ and $1.8 \pm 0.3\,M_\odot$ respectively). These objects show that NSs in binary systems may have a mass larger than $1.64\,M_\odot$.

A natural question that may arise is: why do X-ray binary NSs, which are believed to be the progenitors of binary radio pulsars, have a mass larger than the latter ones ? We have not yet any reliable answer to this question. A first (pessimistic) answer is that the measurements of X-ray NS masses are bad (compare the error bars of the masses of the binary radio pulsars with the ones of the X-ray binaries in Fig. 3 of ref.[29]), and consequently not reliable. Actually it should be noticed that the error bars of the X-ray pulsars do not have the same statistical meaning as the error bars of the binary radio pulsars [31]: they give only the extremum limits of NS masses in the X-ray binary. Consequently $1.4\,M_\odot$ is not incompatible with these masses.

A related question arises naturally: why are the observed masses of millisecond radio pulsars almost identical ? Following the standard model, a millisecond radio pulsar is a recycled NS, spun up by the accretion of mass and angular momentum from a companion. The observed mass and angular velocity are those of the end of the accretion process. Consequently the accreted mass depends on the history of the system and on the nature of the companion. By supposing "per absurdo" that all NSs are born with the same mass, it is difficult to understand why the accreted mass is the *same* for all NSs. A possible answer is that this could result from some observational selection effect. For example, suppose that accreted matter quenches the magnetic field, it is then easy to imagine that the final external magnetic field depends on the mass of the accreted plasma. If the accreted mass is large enough, the magnetic field can be lower than the critical value for which the pulsar mechanism works. On the contrary, if the accreted mass is quite small, the magnetic field is large and the life time of the radio pulsar phase is shorter and consequently more difficult to observe.

4 Detectability

If the nuclear matter EOS is stiff enough and accreting NS in binary systems have a mass large enough for the symmetry breaking to take place, accreting NS are efficient gravitational wave emitters. It is very easy to compute the amplitude of the emitted gravitational waves. By equating the rate of the accreted angular momentum to the rate of radiated angular momentum one obtains [32]

$$h = 1.3 \times 10^{-27} \left(\frac{1 \text{ kHz}}{\nu} \right) \left(\frac{F_X}{10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}} \right)^{1/2} \tag{2}$$

where h is the strain of the gravitational wave, ν the rotation frequency of the source, F_X the X-ray flux received on Earth. Note that the distance of the source does not appear in the above formula. From (2), the signal-to-noise ratio S/N can be easily computed in terms of the observation time T and the sensitivity of the detector B. For the brighest X-ray source, Sco X-1 ($F_X = 2 \times 10^{-7}$ erg cm^{-2} s^{-1}), we obtain

$$\left(\frac{S}{N} \right)_{\text{Sco X}-1} = \left(\frac{0.17}{B/(10^{-23} \text{ Hz}^{-1/2})} \right) \left(\frac{1 \text{ kHz}}{\nu} \right) \left(\frac{T}{1 \text{ day}} \right)^{1/2} \tag{3}$$

From the above formula, we see that one month of observation is sufficient to obtain $S/N = 1$ with a detector of the 10^{-23} Hz$^{-1/2}$ class. This is however a misleading result: in fact, because the frequency of the CW emission is not known, a signal-to-noise of about 7 is required in order to have a detection with a confidence level equivalent to the ordinary 3σ criterium. This means that 2.5 years of observation time with *one* detector are necessary to detect the gravitational radiation emitted by rotating NS. With 3 detectors (e.g. 2 LIGO + VIRGO) the situation appears more favorable: 10 months of observation time instead of 30. Moreover a less naive strategy can be used to couple the 3 detectors; we do not discuss this possibility here.

LTZO objects are also good candidates. The radiation mechanism is analogous to that of the accreting binary sources: the NS forming the core is spun up by the accreted angular momentum from the envelop. The main avantage of these sources is that the mass range of the inner NS spans from the initial mass of the NS ($\sim 1.4\ M_\odot$) up to the critical mass ($\geq 2\ M_\odot$). The drawback is that we do not if these objects exist.

Finally, note that a deformation (ellipticity) as small as $\approx 10^{-8}$ is suffcent to radiate the accreted angular momentum at the Edington mass accretion rate ($10^{-8}\ M_\odot$ yr^{-1}). A question naturally arises: do there exist any other

mechanism able to deform the NS by a such a amount ? No alined magnetic field can do the job. The accreted matter is funelled by the magnetic field onto the crust of the NS, and spreads out on the surface, but magnetic field acts as a magnetic brake for this process. The efficiency of this magnetic braking depends on the conductivity of the plasma and on the strength of the magnetic field. A rough estimation of the typical spreading time τ of the accreted plasma on the surface of the NS gives $\tau \gg 1$ yr. This means that 3-D asymetries can be larger than 10^{-8} for the accreting rate of $10^{-9} - 10^{-8}$ $M_{\odot}$ yr^{-1}. The above encouraging result is correct only if the effecive conductivity of the plasma is equal to the microscopic one. Indeed plasma instabilities can reduce the effective conductivity by orders of magnitude. The most dangerous of them is the instability generating the reconnection of the magnetic field lines. Fortunately, no X or O point exists in the magnetic field configuration. Therefore this kind of instability seems to be excluded. More investigation is needed to clarify this important question (the Authors thank Prof. E. Spiegel and Dr. A. Mangeney for illuminating discussions on this point).

5   CONCLUSION

Accreting NS in binary systems or in LTZOs can be good gravitational CW emitters. Their positions on the sky are known, therefore data can be easily reduced to the solar system barycentric frame and the Doppler shift induced by the motion of the Earth can be properly taken into account. The amplitude of the predicted gravitational waves is large enough to be detected with the 10^{-23} Hz$^{-1/2}$ class of detectors. Positive detection will give to us important informations on the equation of state of the nuclear matter in NS. The proof of existence of the LTZOs will be a major discovery leading to important informations on the stellar evolution during the common envelop phase.

References

1. Landau L., *Nature* **141**, 333 (1938).
2. Thorne K.S., Zytkow A.N., *Astrophys. J.* **212**, 832 (1977).
3. Chandrasekhar S., Phys. Rev. Lett. **24**, 611 (1970).
4. Friedman J.L., Schutz B.F., *Astrophys. J.* **222**, 281 (1978).
5. Friedman J.L., *Commun. Math. Phys.* **62**, 247 (1978).
6. Roberts P.H., Stewartson K. *Astrophys. J.* **137**, 777 (1963).
7. Chandrasekhar S., *Ellipsoidal figures of equilibrium* (Yale University Press, New Haven, 1969).

8. Tassoul J.-L., *Theory of rotating stars* (Princeton University Press, Princeton, 1978).

9. Christodoulou D.M., Kazanas D. Shlosman I., Tholine J.E., *Astrophys. J.* **446**, 510 (1995)

10. Lai D., Rasio F.A., Shapiro S.L., Astrophys. J. Suppl. **88** , 205 (1993)

11. Press W.H., Teukolsky S.A., *Astrophys. J.* **181**, 513 (1973).

12. Miller B.D., *Astrophys. J.* **187**, 609 (1974).

13. Lindblom L., *Astrophys. J.* **438**, 265 (1995).

14. Bertin G., Radicati L.A., *Astrophys. J.* **206**, 815 (1976).

15. Bonazzola S., Gourgoulhon E., in *Astrophysical Sources of Gravitational Radiation (Les Houches 1995)*, eds. J.-A. Marck, J.-P. Lasota, to be published (preprint: astro-ph/9605187).

16. James R.A., *Astrophys. J.* **140**, 552 (1964).

17. Bonazzola S., Frieben J., Gourgoulhon E., *Astrophys. J.* **460**, 379 (1996).

18. Skinner D., Linblom L., *Astrophys. J.*, in press.

19. Pandharipande V.R., *Nucl. Phys. A* **174**, 641 (1971).

20. Bethe H.A., Johnson M.B., *Nucl. Phys. A* **230**, 1 (1974).

21. Friedman B., Pandharipande V.R., *Nucl. Phys. A* **361**, 502 (1981).

22. Haensel P., Kutschera M., Prószyński M., *Astron. Astrophys.* **102**, 299 (1981)

23. Diaz Alonso J., *Phys. Rev. D* **31**, 1315 (1985).

24. Glendenning N.K., *Astrophys. J.* **293**, 470 (1985)

25. Wiringa R.B., Fiks V., Fabrocini A., *Phys. Rev. C* **38**, 1010 (1988)

26. Weber F., Glendenning N.K., Weigel M.K., *Astrophys. J.* **373**, 579 (1991).

27. Salgado M., Bonazzola S., Gourgoulhon E., Haensel P., *Astron. Astrophys.* **291**, 155 (1994).

28. Müller E., in *Astrophysical Sources of Gravitational Radiation (Les Houches 1995)*, eds. J.-A. Marck, J.-P. Lasota, to be published.

29. Thorsett S.E., Arzoumanian Z., McKinnon M.M., Taylor J.H., *Astrophys. J.* **405**, L29 (1993).

30. van Kerkwijk M.H., Bergeron P., Kulkarni S.R., submitted to *Astrophys. J.*(preprint: astro-ph/9606045)

31. Lindblom L., private communication.

32. Wagoner R.W., *Astrophys. J.* **278**, 345 (1984).